Boundary Element Analysis of Cracks Under Compression

Piotr Fedelinski[1, a)]

[1]*Institute of Computational Mechanics and Engineering, Silesian University of Technology, Konarskiego 18A, 44-100 Gliwice, Poland*

[a)]Corresponding author: piotr.fedelinski@polsl.pl

Abstract. The aim of work is analysis of materials with cracks subjected to compressive static loadings. Representative volume elements (RVE) with cracks are modelled using the boundary element method (BEM). The contact forces between crack surfaces are determined using the iterative procedure. Stress intensity factors (SIF) and effective elastic properties for materials loaded by compressive and tensile loadings are compared.

INTRODUCTION

The boundary element method was applied for closure of single cracks by Lee [1], Tuhkuri [2] and Phan et al. [3]. Analysis of effective elastic properties of materials with closed cracks was presented by Nemat-Nasser and Hori [4], Renaud et al. [5] and Liu and Graham-Brady [6]. Fedelinski [7], [8] used the BEM for computation of effective elastic properties and an analysis of SIF for RVE with randomly distributed statically loaded microcracks. The microcracks having the same length, randomly distributed, parallel or randomly oriented were considered. The influence of crack closure on dynamic SIF was demonstrated by Fedelinski in [9].

In the present work materials with multiple cracks are modeled using the BEM. The method requires discretization of external boundaries of the RVE and boundaries of cracks. The variations of boundary coordinates, displacements and tractions are interpolated using shape functions and nodal values. The relations between boundary displacements and tractions are expressed by the displacement and traction boundary integral equations. The direct solutions are displacements and tractions for boundary nodes. The stress intensity factors are computed using the path independent J-integral and the decomposition technique. The displacements of boundaries of the representative volume elements are used to compute average strains and effective Young moduli and Poisson ratios.

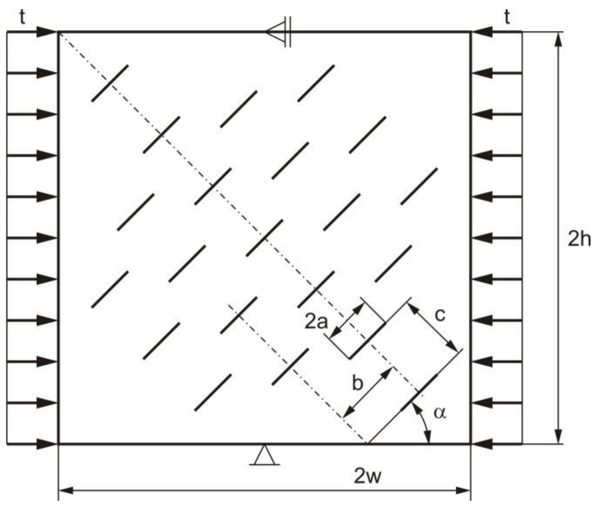
Contact forces are determined using the iterative procedure. In each iteration the relative displacements of pairs of nodes on opposite crack surfaces in the normal and tangential directions are computed. When the opening is negative the crack closure occurs. In this case the pair of nodes is subjected to small normal tractions, which decrease overlapping of crack edges. The iterative process is repeated until the crack opening for the whole crack is positive. The increase of the crack tractions is constant and it is assumed as a fraction of the applied external traction.

NUMERICAL EXAMPLE

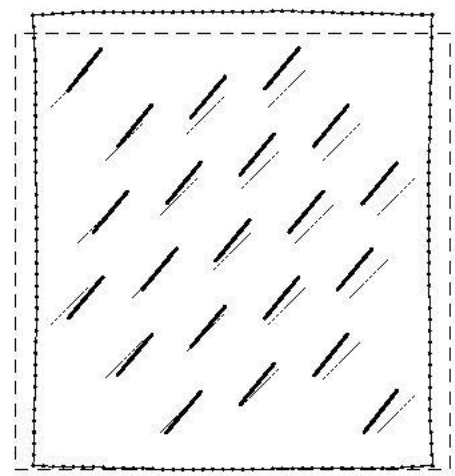
A square representative volume element of width $2w$ and height $2h$ ($h=w$) contains 21 straight cracks of length $2a$ ($a/w=1/8$) inclined at the angle $\alpha=\pi/4$, as shown in Fig. 1a. The distances between the cracks are $b=c=2\sqrt{2}a$. The plate is simply supported and subjected to the compressive loading t in the horizontal direction. The material is linear-elastic, homogenous, isotropic in plane strain conditions and the Poisson ratio is $\nu=0.3$. The plate is divided into 500 boundary elements (80 for the external boundary and 20 for each crack). In Fig. 1b the initial and deformed shapes of the plate are shown.

If contact of crack edges is not taken into account (beginning of the iterative procedure of computation of contact forces) the normalized stress intensity factors for the crack in the center of the plate are: K_I/K_o=-0.701 and K_{II}/K_o=0.541, where the normalizing factor $K_o=t(\pi a)^{1/2}$ (K_I is negative because of overlapping of crack edges) and the normalized effective properties are: the Young modulus E/E_o=0.763 and the Poisson ratio ν/ν_o=0.719, where E_o and ν_o are properties of the continuous material in plane strain conditions. The same absolute values of SIF and effective material properties are for the RVE subjected to tension when all cracks are opened.

When contact of crack edges is considered (end of the iterative procedure of computation of contact forces) the normalized stress intensity factors for the crack in the center of the plate are: K_I/K_o is almost 0 and K_{II}/K_o=0.543 and the normalized effective properties are: the Young modulus E/E_o=0.877 and the Poisson ratio ν/ν_o=1.17. The contact forces have small influence on K_{II}/K_o. The effective Young modulus is increased by 15% and the Poisson ratio by 63% with respect to the properties of the material subjected to tension.



(a)



(b)

FIGURE 1. Representative volume element with inclined cracks under compression:
(a) dimensions and boundary conditions, (b) initial (dashed line) and deformed shape (continuous line)

ACKNOWLEDGMENT

The scientific research is financed by National Science Centre, Poland in years 2016-2019, grant no. 2015/19/B/ST8/02629.

REFERENCES

1. S.S. Lee, Int. J. Fract., **77**, 323-336 (1996).
2. J. Tuhkuri, Int. J. Numer. Meth. Engnr., **40**, 2995-3014 (1997).
3. A-V. Phan, J.A.L. Napier, L.J. Gray, T. Kaplan, Comput. Mech., **32**, 392-400 (2003).
4. S. Nemat-Nasser, M. Hori, *Micromechanics: Overall properties of heterogeneous materials* (Elsevier, Amsterdam, 1999).
5. V. Renaud, D. Kondo, J.P. Henry, Comput. Mater. Sc., **5**, 227-237 (1996).
6. J. Liu, L. Graham-Brady, Int. J. Solids Struc., **100-101**, 151-168 (2016).
7. P. Fedelinski, “Analysis of representative volume elements with random microcracks”, Chapter 17, in *Computational Modelling and Advanced Simulation, Computational Methods in Applied Sciences* 24, edited by J. Murin et al. (Springer Science+Business Media B.V., 2011), pp. 333-341.
8. P. Fedelinski (Ed.), *Advanced Computer Modelling in Micromechanics* (Silesian University of Technology Publisher, Gliwice, 2013).
9. P. Fedelinski, *Boundary Element Method in Dynamics of Deformable Structures* (Silesian University of Technology Publisher, Gliwice, 2016).